

Topics : Limits, Straight Line, Continuity & Derivability, Function, Sequence & Series

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1,2,3,4	(3 marks, 3 min.)	[12, 12]
Multiple choice objective (no negative marking) Q.5,6	(5 marks, 4 min.)	[10, 8]
Subjective Questions (no negative marking) Q.7,8	(4 marks, 5 min.)	[8, 10]

1. $\lim_{x \rightarrow 0^+} \frac{\left(\frac{\pi}{2} - \cot^{-1}\{x\}\right)x}{\operatorname{sgn}(x) - \cos x}$ (where $\{.\}$ and $\operatorname{sgn}(.)$ denotes fractional part function and signum function

respectively) is equal to :

- (A) 2 (B) 1 (C) 0 (D) does not exist

2. Let $\lim_{x \rightarrow 0} \frac{[x]^2}{x^2} = \ell$ and $\lim_{x \rightarrow 0} \frac{[x^2]}{x^2} = m$, then

- (A) ℓ exists but m does not (B) m exists but ℓ does not
(C) ℓ and m both exist (D) neither ℓ nor m exists

3. Least value of function $f(x) = \frac{2\sec^2 x + 2\sec x + 1}{\sec^2 x + \sec x + 5}$ is :

- (A) 2 (B) $\frac{1}{5}$ (C) $\frac{2}{19}$ (D) $\frac{5}{7}$

4. Through the centroid of an equilateral triangle a line parallel to the base is drawn. On this line, an arbitrary point P is taken inside the triangle. Let h denote the distance of P from the base of the triangle. Let h_1 and h_2 be the distance of P from the other two sides of the triangle, then

- (A) h is the H.M. of h_1, h_2 (B) h is the G.M. of h_1, h_2
(C) h is the A.M. of h_1, h_2 (D) none of these

5. Given two straight lines $x - y - 7 = 0$ and $x - y + 3 = 0$. Equation of a line which divides the distance between them in the ratio 3 : 2 (internally) can be :

- (A) $x - y - 1 = 0$ (B) $x - y - 3 = 0$ (C) $y = x$ (D) $x - y + 1 = 0$



6. If $f(x) = [x]$, $g(x) = \begin{cases} 0 & , x \in \mathbb{Z} \\ x^2 & , x \in (\mathbb{R} - \mathbb{Z}) \end{cases}$, then (where $[.]$ is greatest integer function)

- (A) $\lim_{x \rightarrow 1} g(x)$ exists but $g(x)$ is discontinuous at $x = 1$
 (B) $\lim_{x \rightarrow 1} f(x)$ does not exist and $f(x)$ is not continuous at $x = 1$
 (C) $g \circ f$ is continuous function
 (D) $g(x)$ is discontinuous at all integer points

7. Let $f(x) = \operatorname{cosec} 2x + \operatorname{cosec} 2^2 x + \operatorname{cosec} 2^3 x + \dots + \operatorname{cosec} 2^n x$, $x \in \left(0, \frac{\pi}{2}\right)$ and $g(x) = f(x) + \cot 2^n x$.

$$\text{If } H(x) = \begin{cases} (\cos x)^{g(x)} + (\sec x)^{\operatorname{cosec} x} & \text{if } x > 0 \\ p & \text{if } x = 0 \\ \frac{e^x + e^{-x} - 2\cos x}{x \sin x} & \text{if } x < 0 \end{cases} \text{ . Find the value of } p, \text{ if possible to make the function } H(x)$$

continuous at $x = 0$.

8. (i) If $\lim_{x \rightarrow 0} \frac{729^x - 243^x - 81^x + 9^x + 3^x - 1}{x^3} = K(\ln 3)^3$, then find the value of k .

(ii) If $\lim_{x \rightarrow 0} \frac{(1+a^3)+8e^{\frac{1}{x}}}{1+(2+b+b^2)e^{\frac{1}{x}}} = 2$, where $a, b \in \mathbb{R}$, then find the possible ordered pair (a, b) .

Answers Key

1. (A) 2. (B) 3. (B) 4. (C)
 5. (A)(B) 6. (A)(B)(C) 7. $p = 2$
 8. (i) 6 (ii) $(1, 1)$ and $(1, -2)$

